) Se Te – CDS 45 – AM - IV 12/5/18

Q. P. Code: 38142

(3 Hours) **[Total Marks: 80]**

N.B.: 1) Question No. 1 is Compulsory.

2) Answer any THREE questions from Q.2 to Q.6.

3) Figures to the right indicate full marks.

1 (a) Verify Cauchy-Schwartz inequality for u= (2, 1,-3) v= (3, 4,-2). Also find angle between u & v. (5)

(b) If $A = \begin{bmatrix} 2 & 0 & 0 \\ 5 & -1 & 0 \\ 2 & 3 & 3 \end{bmatrix}$ find Eigen values of $A^2 + 6A^{-1} - 3I$. (5)

(c) Evaluate $\int_C \frac{z^3+2Z}{(Z-1)^2} dz$ when C is $|z| = 2$. (5)

(d) Find the extremals of $\int_{x_1}^{x_2} (x + y')y' dx$. (5)

2 (a) Verify Cayley-Hamilton theorem & hence find A^{-1}, where $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$ (6)

(b) Find the extremal of $\int_{x_1}^{x_2} (2xy - y''^2) dx$. (6)

(c) Obtain Laurent's series expansion of $f(z) = \frac{Z+2}{(Z-3)(Z-4)}$ about $z = 0$. (8)

3 (a) Evaluate $\int_0^{1+i} z^2 dz$ along the parabola $x = y^2$. (6)

(b) Show that $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ is derogatory & find its minimal polynomial. (6)

(c) Reduce the following quadratic form into canonical form & hence find it's rank, index, signature &value class $x^2 + 2y^2 + 3z^2 + 2yz + 2xy - 2zx$. (8)

Q.4 (a) Find unit vector orthogonal to both $u = (-6,4,2)$ $v = (3,1,5)$.

(b) Evaluate $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)(x^2+4)} dx$.

(c) Show that matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ is diagonalizable. Also find its diagonal and transforming matrix.

Q.5 (a) Using Rayleigh-Ritz method find solution for the extremal of the functional $\int_0^1 (2xy + y^2 - (y')^2)\, dx$ given $y(0) = y(1) = 0$.

(b) Find an orthonormal basis for the subspace of IR^3 using Gram-Schmidt process where $s = \{(1,0,0), (3,7,-2), (0,4,1)\}$

(c) Find the curve C of given length 'l' which encloses a maximum area.

Q.6 (a) If $A = \begin{bmatrix} \pi & \frac{\pi}{4} \\ 0 & \frac{\pi}{2} \end{bmatrix}$ find $\cos A$.

(b) Check whether the set of all pairs of real numbers of the form $(1, x)$ with operations

$(1, a) + (1, b) = (1, a + b)$ and $k(1, a) = (1, ka)$ is a vector space, where k is real number.

(c) Find the singular value decomposition of $A = \begin{bmatrix} 2 & 3 \\ 0 & 2 \end{bmatrix}$.

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